



京都大学

Research Institute for Mathematical Sciences - Kyoto University

HOMOTOPICAL ANABELIAN GEOMETRY

Reading Seminar - Arithmetic Geometry

—
Second Semester 2019-2020

INTRODUCTION

Following the insight of Grothendieck's letter to Faltings, anabelian geometry in its general form deals with the reconstruction the k -isomorphism class of a variety via the G_k -augmented class of its étale fundamental group. Essentially, the question is thus of isolating arithmetico-geometric invariants that can be recovered in a well-chosen étale homotopy context. A now classical approach was first developed in terms of a (pro)group-theoretic recipe with respect to inertia-decomposition groups by H. Nakamura, A. Tamagawa then S. Mochizuki, with as a result, the *anabelianity of hyperbolic curves* over sub- p -adic fields – see the expository [NTM98].

The goal of this seminar is to present recent anabelian results that ensue the introduction of Artin-Mazur and Friedlander *homotopical étale context* and the consideration of higher étale homotopy groups in arithmetic-geometry by A. Schmidt and J. Stix, see this expository texts¹. This approach leads in particular to a reformulation of Mochizuki's Theorem A in terms of étale homotopy type, and to a higher dimensional anabelian result: *every point of smooth variety over number fields admits an anabelian Artin neighbourhood* [SS16]. Toward these results, we introduce all the required definitions, the classical and modern formalism with some key results of arithmetic homotopy theory (étale homotopy type, model category), we present the key components of classical anabelian geometry (Tamagawa's separation of points and Mochizuki's p -local approach), as well as Hoshi's recent generalization from number to generalized sub- p -adic field for polycurves [Hos19]. A special attention is in particular given on *how classical and homotopical anabelian geometry interact together*. In a broader perspective, homotopical geometry provides new insights in arithmetic geometry for example in terms of stacks and motivic homotopy theory.

This seminar is addressed to Master students, and to PhD and postdoctoral researchers interested in classical and homotopical anabelian geometry. The talks will introduce the required elementary notions of algebraic homotopy, and some fundamental notions of arithmetic geometry (e.g. Jacobian, regular model, Artin good neighbourhood and Serre's "bonté"), which will be illustrated via geometric motivations and key examples.

¹A. Schmidt and J. Stix. "Anabelian geometry with étale homotopy types I & II". in: *Arithmetic Geometry and Symmetries around Galois and Fundamental Groups* (Editors: Collas, B., Débes P., Fried, M.). Vol. 15. 2. Oberwolfach Reports, 2018.

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MAIN REFERENCES.

- [Hos19] Y. Hoshi. “A note on anabelian open basis for a smooth variety”. In: *RIMS Preprint (to appear in Tohoku Math. J.)* 1898 (Sept. 2019).
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PROGRAMME

The following programme is to be understood as a guide for reaching a comprehensive understanding of the proofs of the main result of homotopical anabelian geometry; each speaker will decide on which material to develop, whether or not to focus on complements, as well as the the balance between expository and technicity; talks will be given in *English or in Japanese*. *Please feel free to contact the organizer for access to references or for informal discussion before your talk.*

The terms *spaces* and *pro-spaces* – notations Sp and $\mathrm{pro}\text{-}\mathrm{Sp}$, e.g. Sp_* when pointed – refer to simplicial sets and their pro-category – notations $(\mathrm{pro})\text{-}\mathrm{sSet}$ or $(\mathrm{pro})\text{-}\mathcal{S}\mathcal{S}$; the homotopy category of simplicial sets is denoted by $\mathrm{Ho}(\mathrm{sp})$ or \mathcal{H} , the pro-homotopy category by $\mathrm{pro}\text{-}\mathrm{HoSp}$; morphisms in Homotopy categories are denoted $[X, Y]$.

TALK 0 - ARITHMETIC GEOMETRY: BACK AND FORTH TO ALGEBRAIC TOPOLOGY. Reminders on anabelian problems and results for hyperbolic curves and elementary fibrations; Strom’s model categories on Top – Top & hoTop , homotopy lifting properties, Hurewicz (co)fibrations, homotopy groups, weak homotopy equivalences; computing some π_n ; Quillen’s model category on Top^{CW} – Serre (co)fibrations, CW-approximation, Whitehead Theorem; Simplicial sets – Sing & $|-|$, homology & homotopy, Hurewicz & Whitehead Theorems; Eilenberg-MacLane spaces & Postnikov towers. Towards Verdier’s SGA4 simplicial constructions.

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- [May99] J. P. May. *A concise course in algebraic topology*. Chicago Lectures in Mathematics. University of Chicago Press, Chicago, IL, 1999, pp. x+243. [MR1702278](#).
- [nLb19] A. NLab. “Classical model structure on simplicial sets”. In: *nLab* (Oct. 2019). eprint: <http://ncatlab.org/nlab/revision/classical%20model%20structure%20on%20simplicial%20sets/13>.

TOPIC 1 - HOMOTOPICAL ALGEBRAIC GEOMETRY

The goal of this part is to introduce Artin-Mazur [AM69] and Friedlander-Cox [Fri82; Cox79] étale homotopy and topological types for schemes in terms of Quillen formalism of model categories [Isa01]. We recover and extend familiar arithmetic-geometry results establish a key homotopical lemma, and deal with the essential problem of pointed vs unpointed pro-spaces.

TALK 1 - ÉTALE HOMOTOPY TYPE OF SCHEMES, HIGHER HOMOTOPY GROUPS. (3 hrs)

For a smooth scheme X over k with the Grothendieck étale topology, follow [SS13] to define Artin-Mazur’s and Friedlander’s étale homotopy and topological types $\{X\}_{het}$ and $\{X\}_{et}$ in the pro-categories of pro-(homotopy)spaces $\mathrm{pro}\text{-}\mathrm{HoSp}$ and $\mathrm{pro}\text{-}\mathrm{Sp}$ – see [Fri82] Def. 4.4: pro-category, n -truncation & n -coskeleton, categories $HR(X_{et})$ and $HRR(X_{et})$ (Rigid) Hypercoverings, and homotopy category of Sp ; $\{X\}_{et}$ factorizes $\{X\}_{het}$ Ibid. Prop. 4.5. Define the (higher) homotopy pro-groups of $\{X\}_{et}$ [AM69] 2.2 – also [Fri82] Def. 5.1 – which are generally pro-finite Ibid. Th. 11.1, then the Artin-Mazur completion functor $(-)^{\wedge}$ on $\mathcal{H}_{0,fin}$ Ibid. Th. 3.4.

Two generalisations of SGA1: (1) For X a topological $K(\pi, 1)$, apply the étale-classical comparison theorem Ibid. Th. 12.1 & Cor. 12.10 to show that $\{X \otimes \bar{k}\}_{et}^{\wedge} \simeq \mathrm{Sing}(|X(\mathbb{C})^{an}|)^{\wedge} \simeq K(\pi, 1)^{\wedge}$ where π denotes $\pi_1^{top}(X(\mathbb{C})^{an})$; (2) one has a long homotopy exact sequence for geometric fibrations [Fri82] Th. 11.5, see also proof of [SS16] Prop. 2.8.

Remarks. the pro-group $\pi_1(\{X\}_{et})$ is SGA3's π_1 [AM69] Cor. 10.7 which is SGA1 for X normal scheme; cohomology of $\{X_{et}\}$ for locally Noetherian scheme recovers the étale cohomology groups [Fri82] Prop. 5.9 whose coskeletons are finer than the Čech covering, see Ibid. before 3.3.

TALK 2 - MODEL CATEGORY ON PRO-SPACES AND ARTIN-MAZUR'S. (3 hrs)

Present briefly the principle of a model category on the example of spaces: (co)fibrant objects, weak equivalence, Homotopy category, Quillen functors and equivalence, etc. [GJ09] Chap. I & II. Present Isaksen's model category for pro-Sp in terms of pro-local systems [Isa01] §4 & §6. Present Postnikov towers $\{P_n X \rightarrow P_{n-1} X\}_{n \geq 0}$ - $P_n X$ is a $K(\pi, i)$ $i < n$ and $X = \lim P_n X$. State the Whitehead Theorem [Isa01] Th. 7.3 then Isaksen weak equivalence cohomological criterion [Isa01] Prop. 18.4 and sketch the proof: factorize, retract, lift by Postnikov tower. Discuss the comparison with Artin-Mazur pro-HoSp in terms of Quillen pair Ibid. §8.

TALK 2' - ÉTALE TOPOLOGICAL REALIZATION. (Updated) Included the étale topological realization for schemes of [BS16] (projective model structure on pro-simplicial sheaves, the relative étale homotopy type, comparison with Artin-Mazur construction in Pro-HoSp and Isaksen's more explicit model category.)

TALK 3 - ÉTALE HOMOTOPY TYPES IN ALGEBRAIC GEOMETRY. (3 hrs)

We introduce our first results in-between Algebraic Geometry and Homotopy theory. Introduce coverings of pro-spaces and explain why they are fibration in pro-Sp [SS16] §A.3, then how *homotopy equivalent pro-spaces over a common base can be base-changed along covering* [SS16] §A.6. Deduce the same for étale homotopy types (key homotopical Lemma) [SS16] Lem. 2.2. As application of Talk 2 and of §A.6, prove that *varieties with isomorphic étale homotopy types have isomorphic étale cohomology groups as Galois modules* [SS16] Prop. 2.3. Prove for latter use that for $(X, x) \in \text{pro-Sp}_*$, one has $\pi_n^{top}(X, x) \simeq \lim \pi_n(X, x)$ where $\pi_n(X, x) = [\mathbb{S}^n, X]$ [SS16] Th. A.8.

TALK 4 - POINTED VS UNPOINTED PRO-SPACES. (3 hrs)

We establish that for G_k strongly center-free k -varieties X and Y , their unpointed homotopy morphisms identify to their pointed homotopy $\pi_1(Y_{\bar{k}}, \bar{y})$ -orbits, [SS16] Prop. 2.4 & Cor. 2.5, and:

$$\begin{array}{ccc} \text{Hom}_{\text{Ho}(\text{pro-Sp}) \downarrow (k_{et}, \bar{k}_{et})} [(X_{et}, \bar{x}_{et}), (Y_{et}, \bar{y}_{et})]_{\pi_1(Y_{\bar{k}}, \bar{y})} & \xrightarrow{\cong} & \text{Hom}_{\text{Ho}(\text{pro-Sp}) \downarrow k_{et}} (X_{et}, Y_{et}) \\ & \searrow & \downarrow \text{---} \\ & & \text{Hom}_{G_k}^{out} [\pi_1^{et}(X), \pi_1^{et}(Y)] \end{array}$$

We rely on the topological identification of Talk 3 Th. A.8: Explain the monodromy action of [SS16] §A.2.2, use [Whi78] Chap. III §1 (1.10) to establish Lem. A.9 and Th. A.10 for $\text{Ho}(\text{pro-Sp}_*)$ then state and sketch the proof of Th. A.13 for $\text{Ho}(\text{pro-Sp}_*) \downarrow (B, b)$. Conclude by proving Prop. 2.4.

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TOPIC 2 - ANABELIAN GEOMETRY: FROM CURVES TO HIGHER DIMENSIONS

This part reaches our first homotopical anabelian results, that are a homotopical reformulation of Mochizuki’s anabelian result [Moc99] Th. A/16.5, and a potential generalization to higher dimensional varieties: *for X, Y hyperbolic curves over k sub- p -adic field (resp. X, Y smooth varieties over k number fields with the $(HC_{loc.emb.})$ property, see Talk 6), the map*

$$(-)_{et} : \text{Isom}_k(X, Y) \rightarrow \text{Hom}_{\text{Ho}(\text{pro-Sp})\downarrow k_{et}}(X_{et}, Y_{et})$$

is bijective (resp. a r -split injection, “weak anabelianity”) – see [SS16] Th. 1.1, resp. Th. 1.2. We present the key ingredients of classical anabelian geometry, establish the above results, and study the kernel of the retraction in the case of weak anabelianity.

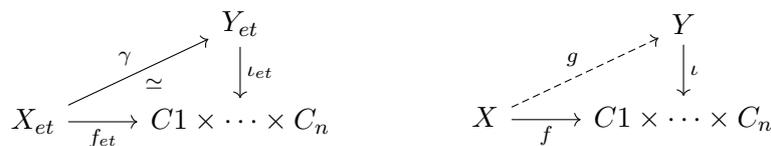
TALK 5 - A HOMOTOPICAL MOCHIZUKI THEOREM. (3 hrs)

First explain Mochizuki’s original Th. A [Moc99] and its context – e.g. sub- p -adic fields have strong center free absolute Galois group Ibid. Lem. 15.8. Sketch the proof by following [Fal98] Th. 14: reduce the assumptions on X, Y and k ; in some extension a point $x \in X(K^+)$ defines a degree prime-to- p line-bundle on $Y(K^+)$ via a geometric section of $\pi_1(J_Y^{(1)})^{(p)}$; similarly for every characteristic open subgroups $H \leq \pi_1(Y \otimes \bar{K})$, one obtains a convergent system of points $y^*(H) \in Y(H)(K^+)$; the dominant morphism follows from a Hodge-Tate argument on the space of global differentials $H^0(-, \omega_{-/k})$. Then gives the definition of an étale $K(\pi, 1)$ scheme [SS16] §2.3 and explain the example of connected smooth curves over a field [SS16] Lem. 2.7(a) with the proof of [Sch96] Prop. 15. State and deduce from Talk 4 the homotopy version of Mochizuki’s Th. A [SS16] Th. 3.2 – assume Prop. A.16

Remark. Details regarding étale $K(\pi, 1)$, including §A.3 and Prop. A.16, will be dealt with in Talk 8; regarding Mochizuki’s result, the speaker can rely on Prof. Hoshi’s seminar and will chose the level of technicity that is adapted to his degree of expertise.

TALK 6 - A WEAK ANABELIAN RESULT. (3 hrs)

Considering k number field and X, Y smooth geometrically connected varieties over k with the $(HC_{loc.emb.})$ property – i.e. that are embedded as locally closed schemes in a product of hyperbolic curves: $\iota : Y \hookrightarrow C_1 \times \dots \times C_n$ –, show that *the map $(-)_{et}$ admits a unique functorial retraction $r : \text{Hom}_{\text{Ho}(\text{pro-Sp})\downarrow k_{et}}(X_{et}, Y_{et}) \rightarrow \text{Isom}_k(X, Y)$ [SS16] Th. 4.7: For $X \rightarrow Y$ given, a commutative diagram in $\text{Ho}(\text{pro-Sp})$ (left below) lifts uniquely in Sch_k (right below) [SS16] Prop. 4.3-4.4 via Talks 3 & 5:*



Give the structure of the proof of the key Prop. 4.6 Ibid. and explain how it boils down to counting points in \mathbb{F}_q ; recover the formula of Prop. 4.1 via Poincaré duality and Lefschetz trace formula; present Tamagawa’s separating point argument [Tam97] Cor. 2.10, or [Fal98] proof of Th. 3; conclude.

Remark. One will assume Lem. 2.9, see Talk 9, and choose the level of details given for the counting formula and Tamagawa’s Lemma.

TALK 7 - CLASS PRESERVATION AND RETRACTION.

(3 hrs)

Our goal is to measure the strong anabelian deficiency of the previous result in terms of the kernel of the retraction, and to establish that for $\gamma \in \text{Ker}(r)$ the induced automorphism $\pi_1(\gamma) \in \text{Aut}_{G_k}^{\text{out}}(\pi_1^{\text{ét}}(X))$ is $\pi_1^{\text{ét}}(X_{\bar{k}})$ class-preserving [SS16] Th. 1.9 (Prop. 5.3/5.5). First, prove Ibid. Prop. 5.1 that $\varphi = \pi_1(\gamma)$ is a normal automorphism by using the generalized Čebotarev density Theorem on a regular model $\mathcal{X}/\text{Spec}(\mathbb{Z})$ and reasoning as in Talk 6 then that φ is trivially of Tate-type, i.e.

$$\varphi(g) = h_g \cdot g^{m_g} \cdot h_g^{-1} \quad \text{with } h_g \in \pi_1^{\text{ét}}(X), \quad m_g \in \widehat{\mathbb{Z}}^*$$

with $m_g = 1$ as G_k -automorphism Ibid. Prop 5.3. If time permits, follow §5.3 to drop the *none rationality* assumption made until there.

Remarks. Give some reminders on Čebotarev density Theorem and regular models.

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- [Tam97] A. Tamagawa. “The Grothendieck conjecture for affine curves”. In: *Compositio Math.* 109.2 (1997), pp. 135–194. [MR1478817](#).

TOPIC 3 - ANABELIAN ZARISKI NEIGHBOURHOODS

We present the key anabelian results for *Artin neighbourhoods (resp. relative open basis) of points in higher dimensional smooth varieties*, i.e. when X is the abutment of successive elementary fibrations of hyperbolic curves with the $HC_{\text{loc.emb}}$ property (resp. with relatively bounded topological data):

$$X = X_n \rightarrow X_{n-1} \rightarrow \dots \rightarrow X_1 \rightarrow X_0 = \text{Spec}(k),$$

first in terms of the homotopical [SS16] Th. 1.5 & Cor. 1.6 over *number fields*, then in terms of the group theoretic [Hos19] for hyperbolic polycurves of strictly decreasing type over *sub- p -adic fields*.

TALK 8 - ANABELIAN CLASSIFYING SPACES FOR PRO-GROUPS.

(3 hrs)

We come back to some Quillen model category constructions to show *the existence of a $K(\pi, 1)$ for pro-groups in the category of pro-spaces* [SS16] §A.3. The goal is to prove Prop. A.16 by following §A.3: property of BG and construction of a fibrant replacement B^*G ; BG is a classifying space in pro-Sp. Using [Isa01] Cor. 7.5 give the characterization of an étale $K(\pi, 1)$ in terms of its classifying morphism Ibid. Cor. A.18. Deduce that morphisms to products of $K(\pi, 1)$ varieties in $\text{Ho}(\text{pro-Sp})$ are given component-wise Ibid. Lem. 2.9 (used in the previous talks).

Application. Let X be a smooth algebraic variety over \mathbb{C} . Using Artin's Theorem [Fri82] Th. 11.6 and [SS16] Pro. 2.8, show that U is an étale and a topological $K(\pi, 1)$, then that $G = \pi_1^{\text{top}}(X(\mathbb{C}))$ is “good” in the sense of Serre [Ser02] I.§2 Exercises. 1 & 2. Following [Qui12] §3.2, show that $\widehat{BG} \rightarrow B\widehat{G}$ is a weak equivalence of pro-spaces iff G is good Ibid. Prop. 3.6.

TALK 9 - STRONG ANABELIAN ZARISKI NEIGHBOURHOODS & RETRACTION. (3 hrs)

Explain and prove the *existence of anabelian Artin neighbourhoods for points in smooth varieties over number fields* [SS16] Th. 1.5 & [SS16] Cor. 1.6. Recall the definition and some examples of strong Artin neighbourhoods [SS16] Def. 6.1 – which in characteristic zero are particular cases of $K(\pi, 1)$. Sketch the proof that *the retraction kernel of Artin neighbourhoods is trivial* [SS16] Th. 6.2 using the lift property of Talk 4, the class-preserving property of Talk 7, and Mochizuki's Th. A. Finally, prove the existence of an anabelian neighbourhoods [SS16] Lem. 6.3, that adapts Artin's seminal construction of good neighbourhoods to rational points [SGA4-3] Exp. XI 3.3.

Also, illustrate some characteristic zero $K(\pi, 1)$ properties: e.g. stability by product for geometrically connected and unibranch varieties – [SS16] Lem. 2.7 (b) via the étale cohomology criterion for weak equivalence [AM69] Th. 4.3 –; morphisms in $\text{Ho}(\text{pro-Sp})$ are given component-wise [SS16] Lem. 2.9 – used in Talk 6.

Remark. Give a preference to a detailed proof of the final result.

TALK 10 - RELATIVE ANABELIAN ZARISKI NEIGHBOURHOODS & POLYCURVES. (3 hrs)

Follow [Hos19] for extending the key result of Talk 9 *from number fields to hyperbolic polycurves over sub- p -adic fields* [Hos19] Cor. 3.4 (i): introduce hyperbolic polycurves of strictly decreasing type (HC_{sdt}) Ibid. Def 1.10 (ii); give the refined existence of a good neighbourhood as in [SGA4-3] Exp. XI 3.3 and Talk 9 Ibid. Lem. 1.11, then show that HC_{sdt} over sub- p -adic fields are relatively anabelian Ibid Th. 2.4 by explaining how a profinite isomorphism $\alpha: \Pi_X \simeq \Pi_Y$ induces isomorphisms on the relative homotopy kernels $\Delta_{X/X_{d-1}} \simeq \Delta_{Y/Y_{d-1}}$ associated to the sequence of parametrizing morphisms $\{X_\bullet \rightarrow S\}$ and $\{Y_\bullet \rightarrow S\}$ Ibid. Lem. 2.3. Explain how the (\dagger_n) group theoretic property [Hos14] Def. 3.10 controls the size of the geometric kernel of $\Pi_Y \rightarrow \Pi_X$ for spreading Mochizuki Theorem A to higher dimensions Ibid. Th. 3.12 – see also Lem. 4.2 (iii).

Remarks. This talk is reserved to an already well-educated anabelian geometer. Beware of the technicity of [Hos14] Prop. 2.4, Prop. 3.2 and Lem. 4.2 (iii).

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- [Hos14] Y. Hoshi. “The Grothendieck conjecture for hyperbolic polycurves of lower dimension”. In: *J. Math. Sci. Univ. Tokyo* 21.2 (2014), pp. 153–219. [MR3288808](#).
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SCHEDULE

The seminar takes place *every two weeks at the RIMS* usually on Monday 13:30-16:30 (room to be confirmed). This allows participants and speakers to assimilate the material and the techniques from the previous talks.



Please feel free to contact the organizer (bcollas@kurims.kyoto-u.ac.jp) for attending and for reserving a slot, or fill in the register form in the link right.

TALKS AND SPEAKERS.

Organizational Meeting & Talk 0: Thursday 17th of October, 13:00 - 15:30, RIMS room 006.

T0	T1	T2	T3	T4	T5	T9	T8	T6	T7	T10
<i>B. Collas</i>	<i>W. Porowski</i> <i>T. Yuji</i>		<i>T. Yuji</i>	<i>B. Collas</i>	<i>K. Higashiyama</i> <i>K. Sawada</i>		<i>XXX</i>	<i>N. Yamaguchi</i>	<i>K. Sawada</i> <i>K. Sawada</i>	
17	11	18	2	16	6	20	3	17	2	16
October	November		December		January		February		March	

NB. Due to some Monday Bank Holidays and the Master defence of some participants, the order of the talks has been updated.

PARTICIPANTS

- (i) Benjamin Collas, RIMS - Kyoto University;
- (ii) Kazumi Higashiyama, RIMS - Kyoto University;
- (iii) Masaoki Mori, Osaka University (TBC);
- (iv) Wojciech Porowski, RIMS - Kyoto University;
- (v) Kenji Sakugawa, RIMS - Kyoto University;
- (vi) Koichiro Sawada, RIMS - Kyoto University;
- (vii) Densuke Shiraishi, Osaka University;
- (viii) Naotake Takao, RIMS - Kyoto University;
- (ix) Naganori Yamaguchi, RIMS - Kyoto University;
- (x) Yu Yang, RIMS - Kyoto University;
- (xi) Tomoki Yuji, RIMS - Kyoto University;

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